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HW0 will have you *manually* evaluate lambda calculus terms

_Elsa_ checks that _reductions_ are valid
Elsa - what is it and how do I use it?
Elsa is implemented as a Haskell package
elsa: A tiny language for understanding the lambda-calculus

elsa is a small proof checker for verifying sequences of reductions of lambda-calculus terms. The goal is to help students build up intuition about lambda-terms, alpha-equivalence, beta-reduction, and in general, the notion of computation by substitution.

[Skip to Readme]

Modules

[Index] [Quick Jump]
Language
  Language.Elsa
  Language.Elsa.Eval
  Language.Elsa.Parser
  Language.Elsa.Runner
  Language.Elsa.Types

Downloads

- elsa-0.2.1.2.tar.gz [browse] (Cabal source package)
- Package description (as included in the package)

Maintainer's Corner

For package maintainers and hackage trustees

- edit package information

Versions [faq]

0.1.0.0, 0.1.0.1, 0.2.0.0, 0.2.0.1, 0.2.1.0, 0.2.1.1, 0.2.1.2

Dependencies

ansi-terminal, array, base (~=4.*), dequeue, directory, elsa, filepath, haskable, json, megaparsec (~=7.0.4), mtl, unordered-containers [details]

License

MIT

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Category

Language

Home page

http://github.com/ucsd-progsys/elsa

Source repo

head: git clone https://github.com/ucsd-progsys/elsa/

Uploaded

by ranjitjhala at Mon Apr 1 19:56:43 UTC 2019

Distributions

NixOS:0.2.1.2

Executables

elsa
How do I run elsa and do the HW?

Options:

1. SSH into ieng6
2. Install stack locally
3. Use online demo
SSH into ieng6

Pros:

● Should have everything installed already
● Standardized and easy for us to help us with

Cons:

● The wifi sucks and you need the internet?
Install stack locally

Pros:

- Everything can be done offline
- We will use Haskell throughout the class, you might want it locally

Cons:

- Installing stack might be annoying
- Unix: should be easy
- Mac: should also be easy with brew
- Windows: ??
- WSL: Didn’t work last time I tried it (like 2 years ago), but might work now?
Online demo

Pros:

● Will “just work”

Cons:

● Very clumsy
Doing the homework

`make test` will check your work

`make turnin` or `git push` turns in the homework

Do not use `=*>` or `=~>` operators!
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What is the lambda calculus

Very simple programming language

Still Turing complete
What is the lambda calculus

It might look silly but...

- Simple **formal model** of programming
- Provides a minimal framework for exploring and reasoning about various PL concepts
- Fundamental to lots of PL research (especially functional programming)
- **Definitely on the exam**
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Syntax

\( x \): Variable

\((\ x \, \to \, M)\): Function abstraction (\( M \) is a lambda term)

\((M \, N)\): Function application (\( M, N \) are lambda terms)

All we can do is declare functions and apply functions!

Functions are \textit{first-class}: We can apply functions to other functions, and a function can return another function.
Syntax

\(\lambda a \rightarrow (\lambda b \rightarrow b)\)  -- Function that takes a parameter “a” and
-- returns a function that takes a param “b”
\(\lambda a \rightarrow \lambda b \rightarrow b\)  -- Syntactic sugar for above
\(\lambda a \ b \rightarrow b\)  -- More syntactic sugar
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Alpha/Beta reductions

Beta step: Calling a function

Alpha step: Renaming a variable inside a function
Beta step

What do we do with \((\ x \rightarrow x\ )\ y\) ?

We can **substitute** \(y\) for \(x\) inside the body of the function: we just get \(y\)

More examples:

\((\ a\ b\ c \rightarrow b\ )\ d\) becomes \((\ b\ c \rightarrow b\ )\)

\((\ b\ c \rightarrow b\ )\ e\) becomes \((\ c \rightarrow e\ )\)

\((\ a\ b\ c \rightarrow b\ )\ d\ e\) becomes \((\ c \rightarrow e\ )\)
In Elsa

1
2 eval beta :  
3     (\ f x -> f (f x)) g 
4
In Elsa

1 eval beta :
2 (∧(f x -> f (f x))) g
3 =b> ∧(x -> g (g x))
In Elsa

1
2  eval beta:
3    (\f x -> f (f x)) g
4      =b> \x -> g (x x)
What if things get weird?

1 eval beta2 :
2 (\x y z \rightarrow z y x) y z x -- uh oh

Can we still perform a beta reduction?
eval beta2 :
\[(\text{x} \rightarrow \text{z} \rightarrow \text{y} \rightarrow \text{x}) \cdot \text{y} \cdot \text{z} \cdot \text{x} = \text{b} > (\text{y} \rightarrow \text{z} \rightarrow \text{y}) \cdot \text{z} \cdot \text{x}\]
This doesn't work!

"y" the argument is a concrete value

"x" and "y" in the function are purely symbolic -- they just refer to the first and second arguments. So when we substitute "y" for "x", we are using the same name to refer to two different things. This doesn't make sense!
We need to be able to rename variables

Alpha steps let you do just this:

```
    eval alpha EQUIV :
        (\x -> x)
    =a> (\y -> y)
    =a> (\z -> z)
```
We use alpha steps to enable beta steps

```
1
2    eval beta2 :
3    (\x y z -> z y x) y z x -- uh oh
4
```
We use alpha steps to enable beta steps

1
2 eval beta2 :
3 ($x\ y\ z\ \rightarrow\ z\ y\ x$) $y\ z\ x$
4 =a> ($x\ a\ z\ \rightarrow\ z\ a\ x$) $y\ z\ x$
5 =b> ($a\ z\ \rightarrow\ z\ a\ y$) $z\ x$
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PA0 Overview

**Goal:** Simplify lambda calculus expressions via alpha/beta steps

**You will need to understand:**

- How to apply alpha/beta steps
- The definitions in each source file

**Be aware:** the lambda calculus is weird! This might take time
PA0 Overview

Each problem will define higher-level concepts with lambda terms:

-- DO NOT MODIFY THIS SEGMENT

let TRUE  = \x y -> x
let FALSE = \x y -> y
let ITE   = \b x y -> b x y
let NOT   = \b x y -> b y x
let AND   = \b1 b2 -> ITE b1 b2 FALSE
let OR    = \b1 b2 -> ITE b1 TRUE b2

Most of these definitions will not make sense on their own!

TRUE and FALSE make no sense without the definition of ITE -- you need to read all the definitions and try to figure out how they work together.
PA0 overview

Elsa also offers a =d> operator

This allows you to replace symbols with their definition -- this is key! Use it early

```plaintext
-- DO NOT MODIFY THIS SEGMENT

let TRUE  = \x y -> x
let FALSE = \x y -> y
let ITE   = \b x y -> b x y
let NOT   = \b x y -> b y x
let AND  = \b1 b2 -> ITE b1 b2 FALSE
let OR   = \b1 b2 -> ITE b1 TRUE b2

-- YOU SHOULD ONLY MODIFY THE TEXT BELOW, JUST THE PARTS MARKED AS COMMENTS

eval not_true :
  NOT TRUE
-- (a) fill in your reductions here
  =d> FALSE
```
However, you can also make the problems too complicated...

If we replace all definitions, we might end up with too much complexity!

Which of these is easier to work with? Why?

```plaintext
```
```plaintext
eval not_true :
    NOT TRUE
    =d> (λ x y -> b y x) TRUE
```
```plaintext
eval not_true :
    NOT TRUE
    =d> (λ x y -> b y x) (λ x y -> x)
```